AH– 1542 CV-19-(S)

(052) M.Sc. (Previous) Mathematics
Examination 2019-20
Compulsory/Optional
Paper-IV
Complex Analysis

Time: Three Hours] [Maximum Marks: 100
[Minimum Passing Marks:-036

Note: Answer all questions. All question carry equal marks.

Q.No. Solve any five Question from the following all question carry equal marks.

1. a. State and prove Morer's Theorm.

   b. Evaluate $\int_C \frac{Z-1}{(Z+1)^2(Z-2)}dz$ Where C:|Z-i|=2

2. a. State and prove Maximum modulus principle

   b. Explain Hadamard's three circles Principle.

3. a. if $w = \left(\frac{z-c}{z+c}\right)^2$ Where c is real and +ve find the area of the Z-plane of which the upper half of the w-plane is the conformal representation.

   b. Define Exponent of convergence and show that Exponent of Convergence σ of a sequence $\{z_n\}$ in given by $\sigma = lim_{n\to\infty} Sup\left\{\frac{logn}{\log(z_n)}\right\}$

4. a. Evaluate $\int_C \frac{z-3}{z^2+2z+5}$dz Where C is a circle

   i.|z|=1 ii. |z+1-i|=2 iii. |z+1+i|=2

5. a. Explain Schwarg's Reflection Principle with Example.

   b. Obtain Jensen's formula.

6. a. Prove that $\sqrt{(\pi)}\ \lceil 2z = 2^{2z-1}\ \lceil(z)\ \left\lceil\left(z+\frac{1}{2}\right)\right.$

   b. Find the Taylor's and laurents series which represents the function.

   $\left(\frac{(z^2-1)}{(z+2)(z+3)}\right)$ in the region.

   i. |Z|<2 ii. 2<|Z|<3 iii.|Z|>3

7. State and prove Montel's Theorm.

8. a. State and prove Inverse function Theorm.

   b. Use Rouche's Theorm to show that the equation $z^2 + 15z + 1 = 0$ has one root is the dise |z|<3/2 and four roots in the annulus 3/2<|z|<2.

9. a. Show that the mapping w=$\sqrt{z}$ transforms the family of circles |z-1| = $\boldsymbol{\lambda}$ into the family of lemniscates
   $|w-1||w+1| = \lambda$.

   b. Prove that $\int_0^{2\pi} \frac{sin^2\theta}{a+bcos\theta}d\theta = \frac{2\pi}{b^2}\{a - \sqrt{a^2-b^2}\}$

10. Stae and prove cauchy –gours at theorm.